MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

15 OCTOBER 2016
2.30 p.m – 4.30 p.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 2 pages excluding the cover page and appendix.

2. Answer **all FOUR** questions from **Section A** and answer only **ONE** question from **Section B**. All questions carry equal marks and the distribution of the marks for each question is given.

3. Write all your answers in the Answer Booklet provided.

Section A: Answer All Questions.

Question 1 (10 marks)

a) Evaluate the following limits.

i. $\lim_{x\to 0}\left(\frac{x^2+3x-\sin 2x}{x}\right)$ (3 marks)

ii. $\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x^2-1}$ (4 marks)

b) Given $f(x)=\begin{cases} ax+b & , \quad x<0 \\ x^2+3a-b & , \quad 0\le x<2 \\ (x-4)^2 & , \quad x\ge 2 \end{cases}$. Find the values of a and b such that the function is continuous for all real numbers.

(2 marks)

Question 2 (10 marks)

a) Evaluate the first and the second derivative for $y=\sqrt{x^2+4x}$. Simplify the answer. (5 marks)

b) Given $x^2+4xy=12$, find

i. the $\frac{dy}{dx}$ by implicit differentiation. (2 marks)

ii. the value of y and $\frac{dy}{dx}$ if $x=1$. (2 marks)

iii. the equation of the tangent line at $x=1$. (1 mark)

Question 3 (10 marks)

Given $y=4x^3-3x^4$.

a) Find the interval(s) of increasing and interval(s) of decreasing. (3 marks)

b) Find the x-coordinates of any local maximum points and local minimum points. (1 mark)

c) Find the interval(s) of concave up and interval(s) of concave down. (3 marks)

d) Find the x-coordinates of inflection points. (1 mark)

e) Sketch the graph. (2 marks)

Continued...

Question 4 (10 marks)

a) Solve the given first order differential equation by the method of integrating factor. $\frac{dy}{dx} + 2xy = x, \quad y(0) = 2.$ (5 marks)

b) Solve the given second order differential equation.
$y''+2y'-8y = 0 \quad y(0) = -1, y'(0) = 2.$ (5 marks)

SECTION B: Answer Only ONE question.

Question 5 (10 marks)

a) Evaluate:

i. $\int 4x(x-2)^2\,dx$ (2 marks)

ii. $\int_0^{\sqrt{\pi}} x\cos 3x^2\,dx$ (3 marks)

b) Use integration by partial fraction to evaluate $\int \frac{2x^2-2x+1}{x^3-x}\,dx.$ (5 marks)

Question 6 (10 marks)

A region, R, is bounded by $y = x^2$ and $y = -x$.

a) Sketch the graph and identify the bounded region R. (2 marks)

b) Find the area of region R. (3 marks)

c) Find the volume of the solid of revolution when the region R is rotated about the y-axis. (5 marks)

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APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$